HARMONIC OSCILLATOR AND THE QUANTUM POTENTIAL

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ABSTRACT

As a matter of fact one of the very interesting issues in Physics is the question of merging quantum world to classical world. Simply, nature has two folds in this sense; quantum and micro universe in which all the principles of quantum mechanics apply, secondly is the large scale universe in which the classical Newtonian mechanics work. According to all theories and approaches discussing the origin of the universe, originally, the universe has been in quantum nature and there was no classical world. This article through a simple example in a well known model (Simple Harmonic Oscillator SHO) demonstrates the merge of the quantum to classical using quantum potential approach.

The Quantum Potential

The Schrödinger equation for any Harmonic Oscillator whether it is right way SHO or upside down USHO, for any potential V(x,t) depends on time can be represented as

$$i\frac{\partial \psi}{\partial t} = -\frac{1}{2m}\frac{\partial^2 \psi}{\partial x^2} + \frac{1}{2}k(t)x^2 \qquad (1)$$

The ground state solution of (1) is given by

$$\psi(x,t) = A(t)\exp[-B(t)x^2] \qquad (2)$$

where *A*(t) and *B(*t) are two complex valued functions and can be found from the boundary conditions. Certainly, in this paper we are not interested in their values as they would not change our conclusion.

According to the quantum potential approach where we write $\psi = Re^{iS}$, R and S are given by

$$R \propto e^{-Re(B)x^2} \qquad (3)$$

$$S \propto -Im(B)x^2 \qquad (4)$$

Here, the Re and Im stand for the real and imaginary parts of the complex function *B*(t).

Then the quantum potential

$$Q = -\frac{1}{2R}\frac{\partial^2 R}{\partial x^2}$$

takes the form [using (3)]

$$Q = \frac{\mathrm{Re}(B(t))}{2}\left(\mathrm{Re}(B(t))x^2 - 1\right) \tag{5}$$

Now this quantum potential becomes negligible i.e. $Q \approx 0$ if the following condition holds

$$|Re(B)| << |Im(B)| \tag{6}$$

This condition has already been introduced by Halliwell [2] to justify that the Wigner function is peaked when (6) holds.

This condition (6) is different for each problem or model for example in the case of USHO discussed in [3] as a Toy Model, the above condition is equivalent to the following; as the time,

$$t \to \infty$$

the quantum description can be replaced by a classical one.

Simple Harmonic Oscillator (SHO)

The Schrödinger equation of SHO is given by

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m}\frac{\partial^2\psi}{\partial x^2} + \frac{1}{2}k\,x^2 \tag{7}$$

We are interested in the form of the function B(t) for both R and S which is given by:

$$B(t) = \frac{\sqrt{mk}}{2}\frac{1}{\cosh(2\phi) - \cos 2\omega(t - t_0)}[\sinh(2\phi) - i\sinh 2\omega(t - t_0)] \tag{8}$$

where φ is a real constant.

We can easily deduce that the system does not reduce to the classical description at t approaches infinity as we had for Upside Down Harmonic Oscillator (USHO) [Aziz, 4]

The quantum potential of this system is given by

$$Q = \frac{\sqrt{mk}}{2m}\frac{\sinh(2\phi)}{\cosh(2\phi) - \cos 2\omega(t - t_0)}[1 - \frac{\sqrt{mk}\sinh(2\phi)x^2}{\cosh(2\phi) - \cos 2\omega(t - t_0)}] \tag{9}$$

And the momentum of the particle in the Ontological Interpretation is given by

$$\pi = \frac{\partial S}{\partial x} = \frac{\sqrt{mk}\,\sin 2\omega(t - t_0)}{\cosh(2\phi) - \cos 2\omega(t - t_0)}\,x \qquad (10)$$

Thus the quantum trajectories are the solution of (10)

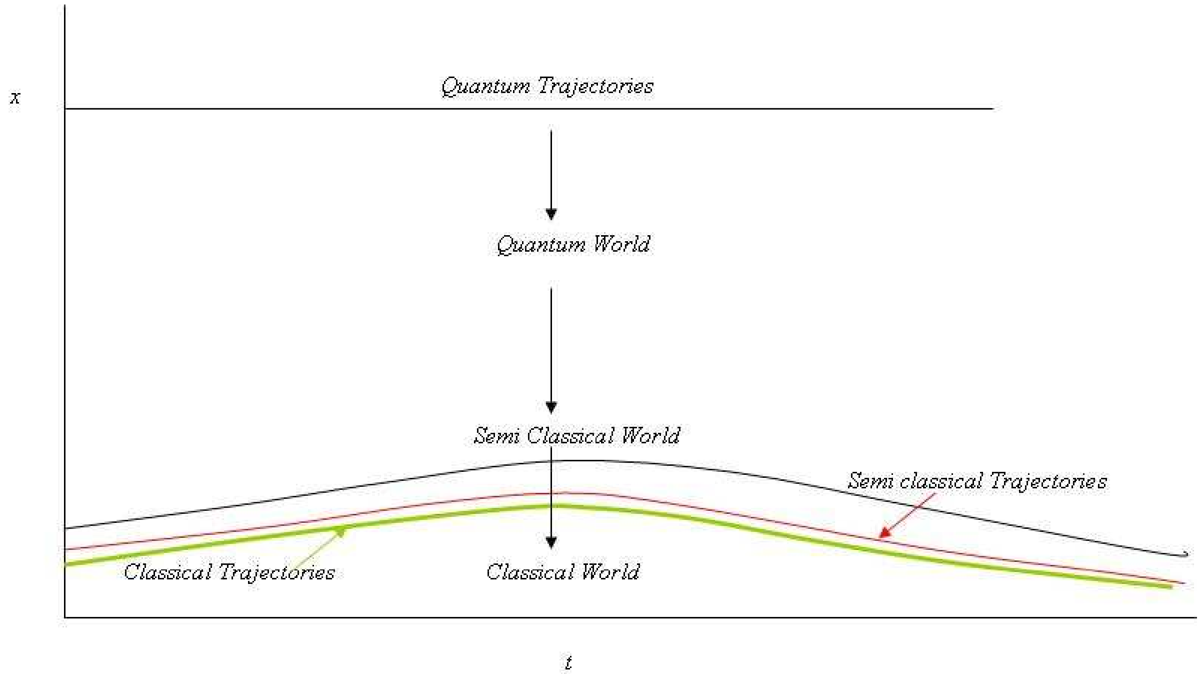
$$x = const.\sqrt{\cosh(2\phi) - \cos 2\omega(t - t_0)} \qquad (11)$$

Now the condition (6) here reduces to the case that $Q \approx 0$ then the Quantum Potential becomes negligible so the system can be described classically.

According to the above statement the trajectories (11) become the classical trajectories when $\varphi \approx 0$ or $Q \approx 0$ i.e.

$$x = A \sin \omega(t - t_0) \qquad (12)$$

The following Figure shows how the quantum trajectories become the classical path when Q approaches zero or becomes negligible.



Conclusion

In this paper I have shown the use of the Ontological Approach to study the classical limit of the well known Simple Harmonic Oscillation. A similar study using statically approach has already been used to demonstrate the way the quantum world would become a classical one [2]. The method I used here has a number of advantages; the pattern of the trajectories in either quantum or classical is very clear. In classical mechanics we already talk about paths and particle trajectories, so if a similar language is used to develop from the quantum world, things would look more understandable. The notion of statistics usually cannot be accommodated easily in classical world. Transition from quantum world to classical one would look vague. That is why my method has more advantages to view such picture.

References

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